Radioactivity & Radionuclide Production

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- Parent-daughter mixtures and radionuclide generators
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- Production mechanisms
  - neutron activation
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Radioactivity (ABR core study guide 17.c.i(a)-(b))

Consider a sample of radioactive material. The fractional change in the number of radioactive atoms during a short time, Δt, is linearly related to the time interval. The constant of proportionality is called the **decay constant** for the radionuclide:

$$\frac{\Delta N}{N} = -\lambda \Delta t$$

$$Activity\,(Bq) = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N,$$ where 1 Bq = 1 decay/second

$$Activity\,(Ci) = \lambda N / (3.7x10^{10})$$ (Ci = Curies)

1 mCi = 3.7 x 10^7 dps = 37 MBq

Exponential Decay (ABR core study guide 17.c.i(b) and 17.c.iii)

Use calculus to solve for the number of radioactive atoms remaining in the sample as a function of time. (Integrate both sides of equation.)

$$\frac{dN}{N} = -\lambda dt$$

$\ln(N) - \ln(N_o) = -\lambda t,$

where N_o = initial number

$\ln(N/No) = -\lambda t$

$N(t) = N_o e^{-\lambda t}$

Thus:

$A(t) = A_o e^{-\lambda t}$

Half-Life (ABR core study guide 17.c.i(b) and 17.c.iii)

The half-life is the time required for the radioactivity to decay to half of its initial value:

$$\frac{1}{2} A_o = A_o e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln(1/2) = -\lambda t_{1/2}$$

$$\ln(2) = \lambda t_{1/2}$$

$$t_{1/2} = \ln(2)/\lambda \sim 0.693/\lambda$$

(The **average** lifetime $= 1/\lambda$.)

Commonly used radioisotopes

Radionuclide	Half-life	Decay constant
Technetium-99m	6.02 h	0.1151 h^{-1}
Fluorine-18	110 m	0.0063 m^{-1}
Iodine-123	13.27 h	0.0522 h^{-1}
Iodine-131	8.02 d	0.0864 d^{-1}
Nitrogen-13	10 m	0.0693 m^{-1}
Carbon-11	20 m	0.03465 m^{-1}
Zirconium-89	78.4 h	0.0088 h^{-1}

Specific Activity and Tracer Principle (ABR core study guide 17.c.ii)

The **specific activity** is the ratio of the radioisotope's activity to the total mass of the same element or compound (Bequerels per gram).

The **carrier-free specific activity (CFSA)** is the highest possible specific activity of a radionuclide, i.e. with no "cold" carrier present.

$CFSA(Bq/g) \sim 4.8 \times 10^{18}/(A\ t_{1/2})$, where

A = mass number of the radionuclide or compound,

$t_{1/2}$ = half-life in days.

(Note: Easier to get high specific activity for short half-life nuclides.)

$CFSA(Ci/g) \sim 1.3 \times 10^{8}/(A\ t_{1/2})$, in old units.

Specific Activity and Tracer Principle (ABR core study guide 17.c.ii)

Requirements of ideal tracers:

1. Tracer behavior should be as close as possible to that of the natural substance

2. Mass of tracer should not alter underlying physiologic process

   - rule of thumb: mass of tracer < 0.01 x mass of endogenous compound

3. Specific activity high enough to permit imaging or blood counting without violating conditions 1 and 2.

4. Any isotope effect should be negligible (or quantitatively predictable).

Example: What is the mass of 10 mCi of $H_2{}^{15}O$? (typical activity injected for PET)

- $t_{1/2}$ of ^{15}O is 2 minutes = 0.001389 days, and the molecular weight of $H_2{}^{15}O$ is 17.
- CFSA = 1.3×10^8 / (17 x .001389) = 5.5×10^9 Ci/g.
- A more typical specific activity might be 10% of the CFSA ~ 5.5×10^8 Ci/g.
- 10 mCi = 0.01 Ci, so mass = 0.01 Ci / 5.5×10^8 Ci/g = 18.2×10^{-12} g = 18.2 pg.
- **18.2 pg -- diluted throughout the whole body -- is clearly a trace amount.**

Radionuclide Equilibrium (Parent-Daughter Mixtures) (ABR core study guide 17.c.iv)

Complicated situation: parent radionuclide gives rise to new daughter radioactivity, even as the daughter's activity decays.

Activities are described completely by **Bateman equations.**

Approximations of interest:

- **secular equilibrium** ($T_p >> T_d$), e.g. Ra-226->Rn-222

$$A_d(t) = A_p(0)\left(1 - e^{-\lambda t}\right)$$ (1620 y >> 4.8 d)

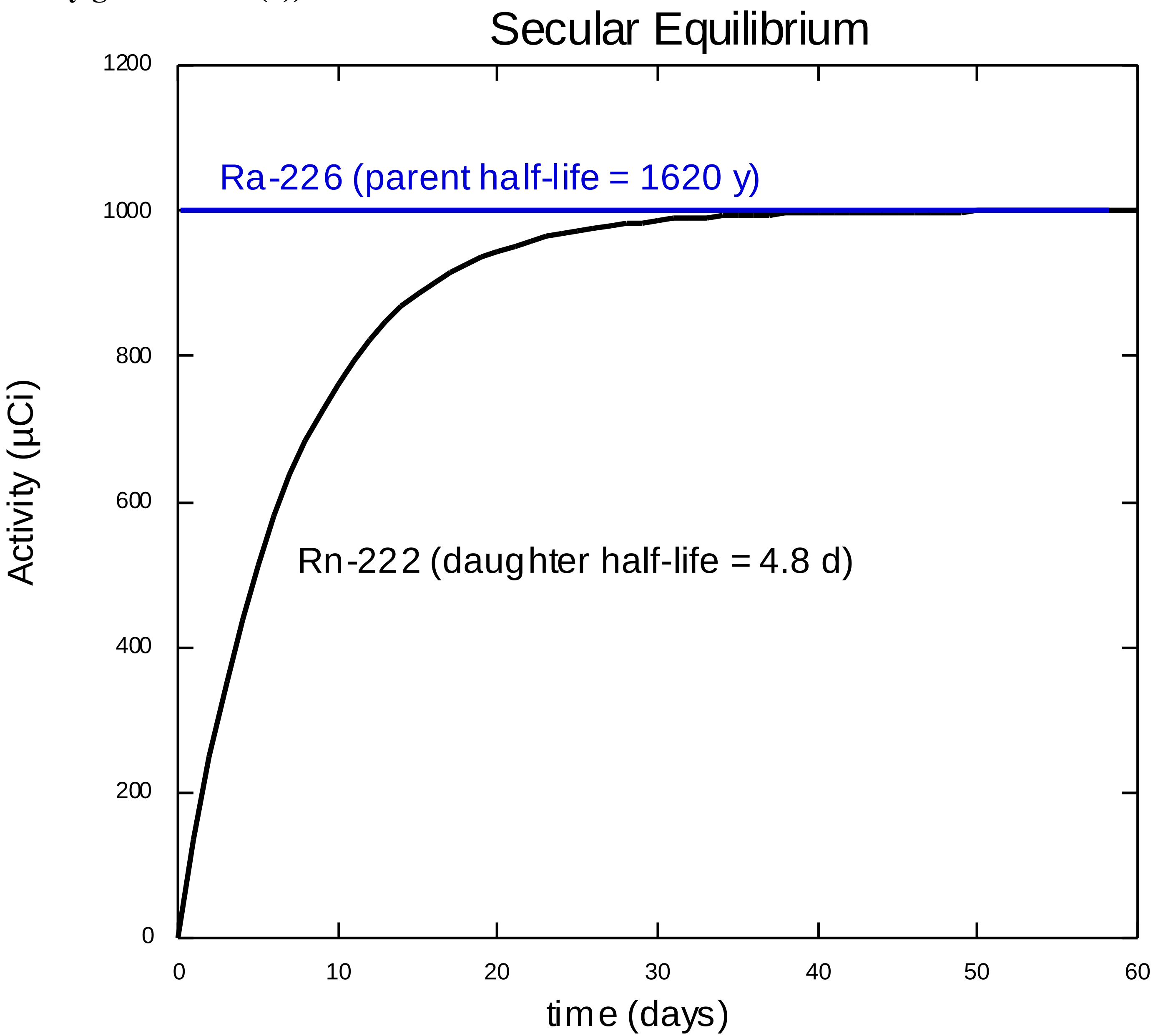
- **transient equilibrium** ($T_p > T_d$), e.g. Mo-99 -> Tc-99m

$$A_d/A_\mathrm{p} = \frac{T_p}{(T_p - T_d)}$$ (66 h > 6 h)

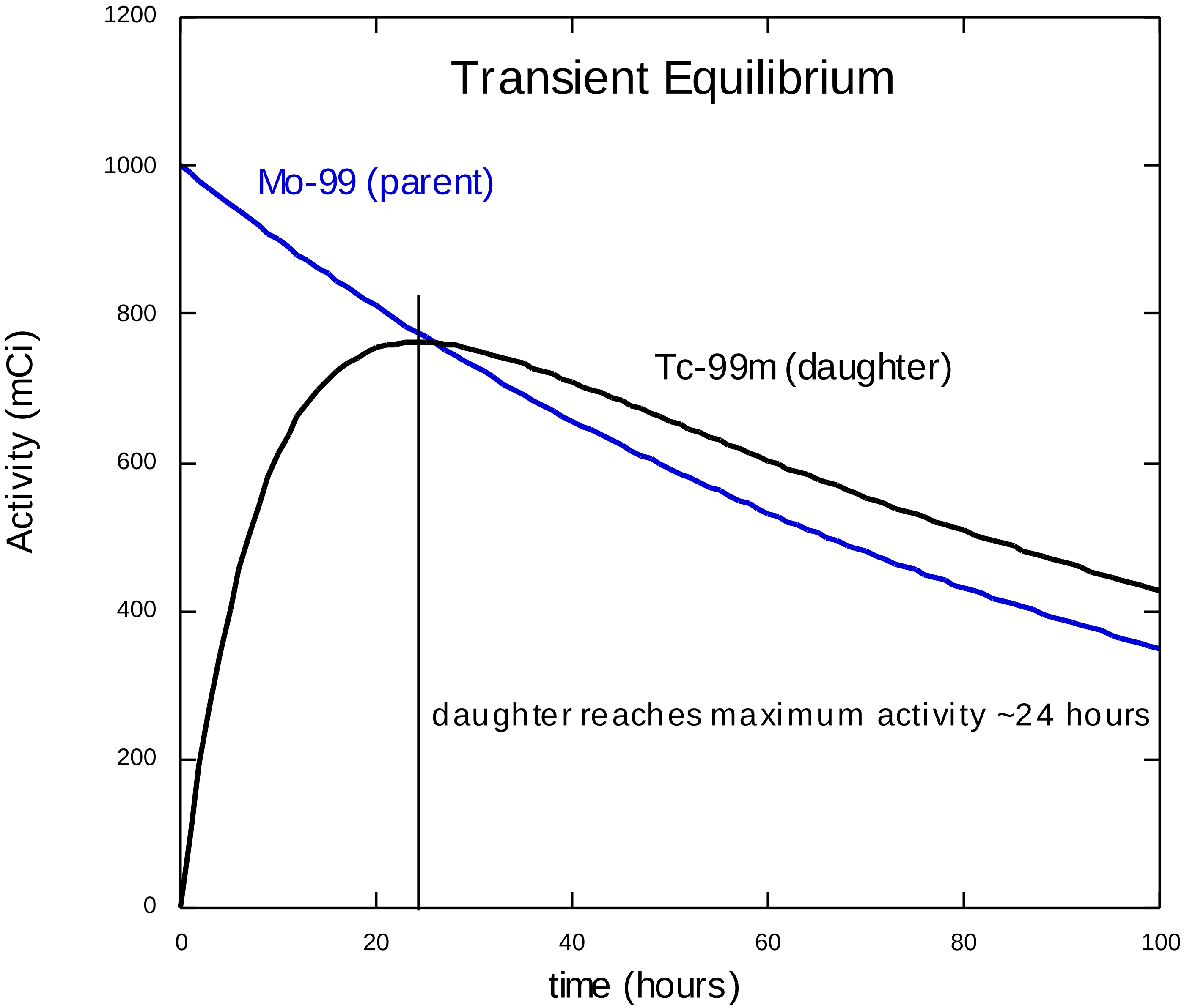
- **no equilibrium** ($T_d > T_p$), e.g. Te-131m -> I-131

parent goes away, daughter decays (30 h < 8 d)

(ABR core study guide 17.c.iv(a))



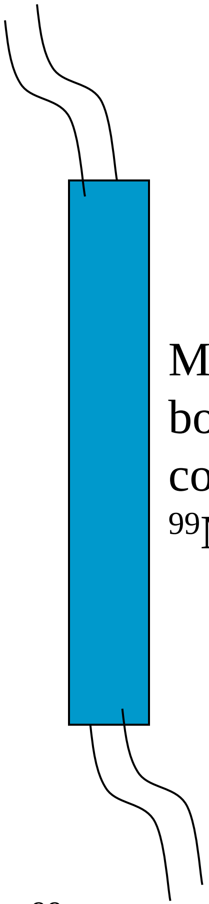
(ABR core study guide 17.c.iv(a))



(ABR core study guide 17.c.iv(b))

Mo-99 / Tc-99m Generator
(transient equilibrium)

inject saline



Molybdate ions, $^{99}MoO_4^{2-}$ bound to an alumina (Al_2O_3) column.

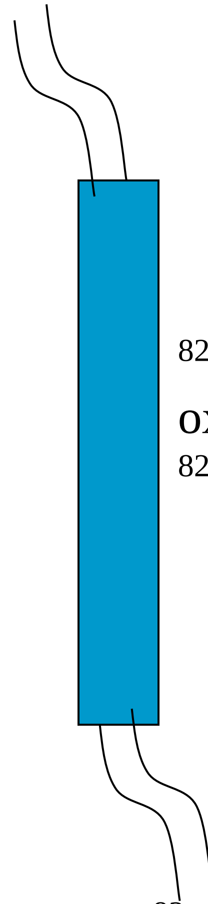
^{99}Mo $t_{1/2}$=66 h; ^{99m}Tc $t_{1/2}$=6 h

Collect $^{99m}TcO_4^-$ (pertechnetate) in eluate

- Check for alumina (<10 µg/mL) and Mo-99 breakthrough (<0.15 µCi Mo-99 / mCi Tc-99m)

Sr-82 / Rb-82 Generator
(secular equilibrium)

inject saline



^{82}Sr adsorbed on a stannic oxide (SnO_2) column.

^{82}Sr $t_{1/2}$ = 25.6 d; ^{82}Rb $t_{1/2}$ = 75s

Collect $^{82}RbCl$ in eluate

- Check Sr-82<.02kBq and Sr-85<.2kBq per MBq of Rb-82 administered

(ABR core study guide 17.d.i)

Radionuclide Production: Neutron Activation

- In a nuclear reactor, fission reactions break apart U-235 into multiple "fission fragments" and release many neutrons.

- The neutrons can be used to irradiate various targets, which are placed inside the reactor. The targets absorb neutrons to become "activated".

1. P-32 production (14.3 day $t_{1/2}$)

$$^{31}P(n,\gamma)^{32}P$$

2. Cr-51 production (27.8 days $t_{1/2}$)

$$^{50}Cr(n,\gamma)^{51}Cr$$

(ABR core study guide 17.d.ii)

Radionuclide Production: Fission Byproducts

1. Generator Production of Tc-99m (many uses)

In nuclear reactor:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{99}_{42}Mo + ^{134}_{50}Sn + 3^{1}_{0}n + \gamma + \sim 200\,MeV$$

In generator:

$$^{99}_{42}Mo \xrightarrow{\beta^{-}(66h)} {}^{99m}_{43}Tc \xrightarrow{\gamma(6h)} {}^{99}_{43}\mathbf{Tc}$$

Radionuclide Production: Fission Byproducts

2. I-131 Production (used for thyroid imaging + therapy):

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{131}_{53}I + {}^{102}_{39}Y + 3\,{}^{1}_{0}n + energy $$

$$ {}^{131}_{53}I \xrightarrow{\beta^{-}(8d)} {}^{131}_{54}Xe $$

3. Xe-133 is another fission byproduct (lung vent imaging)

Radionuclide Production: Cyclotron Produced

1. Thallium-201:

$$^{203}\mathbf{Tl}(\mathbf{p},3\mathbf{n})^{201}\mathbf{Pb}$$

$EC(9.4h)$

$$^{201}Tl$$

$EC(73h)$

$$^{201}Hg$$

Image mercury x-rays

- myocardial perfusion
- tumor imaging

Radionuclide Production: Cyclotron Produced

2. I-123:

- thyroid imaging
- MIBG imaging

$$^{124}\mathbf{Xe(p,2n)}^{123}\mathbf{Cs}$$

$\beta^{+}(5.87m)$

$$^{123}Xe$$

$EC(73h)$

$$^{123}I$$

$EC(13.2h)$

$$^{123}Te$$

(ABR core study guide 17.d.iii)

Radionuclide Production: Cyclotron Produced

3. In-111:

$$^{111}\mathrm{Cd}(p,n)^{111}\mathrm{In}$$

$$^{111}\mathrm{In} \xrightarrow{EC(67h)} {}^{111}Cd$$

- octreotide
- WBCs

4. Ga-67:

$$^{66}\mathrm{Zn}(d,n)^{67}\mathrm{Ga}$$

$$^{67}\mathrm{Ga} \xrightarrow{EC(78h)} {}^{67}Zn$$

- lymphoma and
- infection imaging

(ABR core study guide 17.d.iii)

Radionuclide Production: Cyclotron Produced PET Tracers

5. F-18 (e.g., FDG): $^{18}O(p,n)^{18}F$ (110min)

6. N-13 (e.g., ammonia): $^{16}O(p,\alpha)^{13}N$ (10min)

7. O-15 (water) $^{14}N(d,n)^{15}O$ $^{15}N(p,n)^{15}O$ (2.0min)

8. C-11 (e.g, acetate) $^{14}N(p,\alpha)^{11}C$ (20.4min)

Questions to be answered

1. The activity of a given sample of a radionuclide depends on
a) only the number of radioactive nuclei present in the sample
b) only on the half-life of the radionuclide
c) both the number of nuclei and their half-life
d) the energy difference between the excited state and the ground state

2. When can patients who claim to have a severe “iodine allergy” have an ^{123}I thyroid scan?
a) no restrictions
b) only after taking KI to block thyroid uptake of ^{123}I
c) never
d) only in an emergency

3. The decay constant of a radionuclide
a) is inversely related to its half-life, i.e., ln(2)/half-life
b) is a direct measure of the radioactivity of the radionuclide
c) describes the mean number of gamma photons emitted per decay
d) is linearly related to the total energy emitted by all particles

4. The specific activity of a given sample of a radiopharmaceutical
a) depends only on the half-life of the radionuclide
b) is highest when the sample contains mostly ‘cold’ (non-radioactive) molecules
c) Is lowest when the sample is 100% carrier-free specific activity
d) is usually highest for radionclides with very short half-lives

Questions to be answered (continued)

5. The parent nuclide in a Sr-82 / Rb-82 generator is Sr-82 (25.6 d half-life) and the daughter is Rb-82 (75 s half-life). These two radionuclides on the column are in a state of
a) secular equilibrium
b) transient equilibrium
c) no equilibrium
d) high anxiety

6. After a Mo-99 / Tc-99m generator is eluted
a) the generator should not be eluted again for at least 10 minutes
b) the generator will yield maximum Tc-99m activity if the next elution is 6 hours later.
c) the eluate should only be tested for alumina breakthrough (<1 g/mL)
d) the eluate should be tested for alumina breakthrough (<10 μg/mL) and for Mo-99 breakthrough (<0.15 μCi Mo-99 / mCi Tc-99m)

7. Radionuclides produced in a cyclotron are most likely to decay
a) only by electron capture
b) by either positron emission or electron capture
c) by emission of one or more negative beta-particles
d) by emission of an alpha particle

8. The Mo-99 used for Tc-99m generators is mostly produced
a) in a nuclear reactor by neutron activation
b) in a cyclotron by accelerating deuterons
c) in a nuclear reactor as a byproduct of nuclear fission
d) as a byproduct of the decay of Xe-133